Using Precedent Information in Fuzzy Expert Systems

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Abstract—Many pattern recognition tasks are connected with expert information, which can be expressed in terms of linguistic rules. A theory of fuzzy sets presents one of the most successful ways of using these rules. However, in this case, two main problems appear that cannot be completely solved by experts in some problem domains: fuzzy set formation and fuzzy rule generation. A possible solution based on the use of precedent information is proposed in this paper.

INTRODUCTION

In many pattern recognition tasks, in addition to a learning sample, there is some a priori information about the process under study. This knowledge (expert information) can be expressed as qualitative relations between object features and a predictable value. A theory of fuzzy sets [1] provides a convenient means for representing this information. Most difficult in this approach is the determination of membership functions and the formation of a system of rules of inference. Often, these issues are related to the expert’s competence. Unfortunately, there are some problem domains where experts are not able to solve the problem in full measure. These bottlenecks of the theory of fuzzy sets hamper the employment of fuzzy expert systems. The solution can be found in using precedent information. In this paper, one possible way of using it during construction of a fuzzy expert system is presented.

DESCRIPTION OF EXPERT INFORMATION

Suppose we have a learning sample as initial information. We use the concept of a linguistic variable for representing expert information [1]. The general form of the expert statement takes on the following form:

\[
\text{IF } X_i \in A^i_{k_i}, \ldots, X_j \in A^j_{k_j}, \text{THEN } Y \in B_k. \quad (1)
\]

Here, \(X_i\) is the value of the \(i\)-th feature, \(A^i_{k_i}\) is a fuzzy set with the meaning of a linguistic variable value, \(Y\) is a predictable variable, and \(l\) is the order of the sentence. Thus, an expert needs only define qualitative relations, such as “IF the average temperature is high and precipitation is normal, THEN the crop will be large.” This form of a sentence is close to natural language and allows experts to formulate their thoughts more freely.

Usually, for constructing such systems, an expert must determine the form and the size of fuzzy sets \(A^i_{k_i}\).

Note that, in most cases, this task is very complicated. In practice, however, an expert can indicate the approximate boundaries between the fuzzy sets of a feature. We use an isosceles triangle and trapezium as the forms of fuzzy sets (see Fig. 1).

Let \(\mu_A(x) : X \rightarrow [0,1]\) be a membership function of a fuzzy set \(A\).

Definition 1. The coating of interval \([a, b] \subset R\) is a collection of fuzzy sets \(\{A_i\}_{i=1}^n\) such that \(\forall x \in [a, b] \exists i \in \{1, \ldots, n\} : \mu_{A_i}(x) > 0\). If, in addition, \(\mu_{A_i}(x) > \alpha\), then the coating is \(\alpha\)-significant.

Let \(\{[a_i, a_{i+1}]\}_{i=1}^n\) be a partition of interval \([a, b]\).

Definition 2. Regular coating of interval \([a, b]\) for partition \(\{[a_i, a_{i+1}]\}_{i=1}^n\) is a coating \(\{A_i\}_{i=1}^n\) such that

(i) \(\mu_{A_i}(a_{i+1}) = \mu_{A_{i+1}}(a_i), \forall i = 1, n-1\),

(ii) \(\forall i \in \{1, \ldots, n\} \exists x^*_i \in [a_i, a_{i+1}] : \mu_{A_i}(x^*_i) = \max_x \mu_{A_i}(x)\).

Obviously, the regular coating is \(\alpha\)-significant for \(\alpha = \min_{i} (\mu_{A_i}(a_i), \mu_{A_i}(a_{i+1}))\).

\[\mu_A(x)\]

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Fig. 1. Forms of fuzzy sets.

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Suppose also that not only the rules of inference are known but information about their weights is likewise known. These weights can be specified by an expert or obtained from the learning sample. Let us denote the weight of the \(i\)th rule by \(w_i\). Then, Eq. (4) is transformed to

\[
\mu_R(x, y) = \sum_{i: \text{res} R_i = B} w_i \mu_{R_i}(x, y).
\]  

(5)

Let \(q\) fuzzy sets be determined in the space of the predictable value. We denote the median of the \(k\)th fuzzy set by \(\text{med}B_k\); i.e.,

[\text{med}B_k = \frac{a_k + a_{k+1}}{2}].

Then, the point forecast can be found as

\[
\hat{z} = \frac{\sum_{k=1}^{q} \text{med}B_k \mu_{R_k}(x, \text{med}B_k)}{\sum_{k=1}^{q} \mu_{R_k}(x, \text{med}B_k)}.
\]  

(6)

Thus, we have the following model of forecast calculation:

\[
M(\{a_i\}_{i=0,k+1}^{n,k+1}, \{\alpha_i\}_{i=0,k+1}^{n,k+1}, \{\beta_i\}_{i=0,k+1}^{n,k+1}, \{R_i\}_{i=1}^{m}, \{w_i\}_{i=1}^{m}).
\]  

(7)

Let us denote precedent information by \(\{\hat{x}_k, \tilde{y}_k\}_{k=1}^{\hat{q}}\). Then, we can find parameters \(\{\alpha, \beta\}\) of the model \(M\) by optimization on precedents with the following quality functional:

\[
J = w \sum_{k=1}^{\hat{p}} (\hat{y}_k - \tilde{y}_k)^2 + (1 - w) \sup_{k} |\hat{y}_k - \tilde{y}_k|.
\]  

(8)

where \(\hat{y}_k = M(\ldots)(x_k)\) and \(w \in [0, 1]\) is the parameter of optimization corresponding to the requirements on the expert system.

To reduce the amount of parameters for optimization, we assume that \(\alpha\) and \(\beta\) coefficients are related to features rather than to separate sets. Then, it is possible to take \(\alpha_1' = \ldots = \alpha_{k+1}' = \alpha', \beta_1' = \ldots = \beta_k' = \beta'\). In addition, formula (6) gives us the opportunity of making no optimization in the space of answers. Therefore, finally, the model of expert system (7) transforms to

\[
M(\{a_i'\}_{i=0,k+1}^{n,k+1}, \{\alpha'\}_{i=0,k+1}^{n,k+1}, \{\beta'\}_{i=1}^{n}, \{R_i\}_{i=1}^{m}, \{w_i\}_{i=1}^{m}).
\]  

(9)

\[\]
GENERATION OF THE RULES OF INFERENCE

One of the main problems of fuzzy expert systems is to obtain rules from experts. Therefore, it is necessary to be able to extract these rules from input data automatically.

Definition 4. The representativeness of a rule is the following value:

\[ \text{rep}(R) = \frac{1}{p} \sum_{j=1}^{p} \min \left( \mu_{A_{i_j}}(\hat{x}_{j_1}), \ldots, \mu_{A_{i_j}}(\hat{x}_{j_l}) \right), \]

where \( \{ \hat{x}_k, \hat{y}_k \}_{k=1}^{p} \) is precedent information and the rule of inference takes on the form of (1).

Definition 5. The effectiveness of the rule of inference is the following value:

\[ \text{eff}(R) = \frac{\sum_{i=1}^{p} \min \left( \mu_{A_{i_j}}(x_{j_1}), \ldots, \mu_{A_{i_j}}(x_{j_l}), \mu_{B_{i_j}}(\hat{y}_j) \right)}{\text{rep}(R)}. \]

Then, the common scheme of rule generation will be the following:

Step 1. Construct all possible rules of the first order from available fuzzy sets, i.e., rules of the type “IF \( X_{i_1} \in A_{i_1}^{j_1} \), THEN \( Y \in B_{i_j}^{j_l} \)”. Calculate the representativeness of each rule and reject the rules with representativeness lower than some specified threshold \( C_1 \). For the rest of the rules, calculate effectiveness. The rules with effectiveness higher than the threshold \( C_2 \) are the desired rules of inference.

Step 2. Use the rules with representativeness higher than \( C_1 \) and effectiveness lower than \( C_2 \) to construct rules of the second order for increasing effectiveness. Add the obtained rules with representativeness and effectiveness higher than the corresponding thresholds to the set of inference rules.

Step \( i \). Combine the rules of the \((i-1)\)th order with \( \text{rep}(R) > C_1 \) and \( \text{eff}(R) < C_2 \) into the rules of the \( i \)th order.

The premises of each combined rule lies in the union of the premises of any two combined rules (see Fig. 3).

Add the rules with the representativeness \( \text{rep}(R) > C_1 \) and effectiveness \( \text{eff}(R) > C_2 \) to the set of inference rules. Reject all rules with representativeness lower than the specified thresholds. If the set of remaining rules of the \( i \)th order is not empty, go to the next step, otherwise end.

The effectiveness values of the obtained rules can be used as their weights.

CONCLUSIONS

In this paper, we present an approach to the description of expert information based on the concepts of fuzzy logic. One of the main advantages of this approach is the following: an expert no longer needs to give full information about the form and size of fuzzy sets used in the system, it is only required to make a partition of the feature space. Also, we describe here a way of generating rules from input data without experts. This method substantially simplifies the use of expert systems and broadens the area of their application.

REFERENCES
