ON ONE METHOD OF PROBABILISTIC SIGNALS FILTRATION

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In this paper a method for solving filtration task of special kind is suggested. We consider a particular task of significant noise removal, connected with partial signal's losses, capturing hardware failures, etc. The algorithm is based on probabilistic approach to filtration. The efficiency of proposed method is demonstrated on the task of preliminary processing of pupillograms – reaction of eye's pupil on light flare.

Introduction

The task of digital signal processing appears in a wide range of practical tasks of data analysis, forecasting and decision making. In this paper there considered the task of special noise removal, connected with partial signals losses. The existing methods of signals filtration allow, in principle, to remove noise. Nevertheless, they use some kinds of signals smoothing [2]. As a result local peculiarities of signals are lost, which at the same time may concern main goals of consequent research. So there is a need to develop and implement such methods of filtration, which, from the one hand, remove significant noise, but, from the other hand, influence a little on pure signal. The possible way for such processing is probability theory and, in particular, maximal likelihood principle. One can suppose input signal as realization of stochastic process with some evident properties in case of considered task and thus estimate degree of plausibility of the point's presence in useful signal. Following further, the point with maximal plausibility is stayed for output signal, but the points which precede it, are eliminated as noise. Notice that analogous ideas underlie at structural signal analysis and with hidden markov models construction [1]. However, this paper considers application of this methodology to tasks of signals filtration.

In the next section the construction of discrete probabilistic filter is proposed. Then section 3 focuses on solving of one particular problem which appears within application of proposed approach – the search of first point of useful signal. In section 4 results of practical experiments are given and comparison with other methods of filtration is considered.

The development of discreet probabilistic filter

Consider the following task. Let we have some noisy signal, which is known at time moments $t_1,\dots,t_T$. We will take into account the case of uniform grid, i.e. when $t_i = t_1 + h(i-1)$ and denote the input signal as $x[i]$, where $i = 1, 2,\dots, T$. The task is to construct filter, which outputs signal $y[i]$ in such a way, that filtered signal consists of no significant noise, connected with partial losses during capturing. In case of noise absence, the input signals shouldn't be distorted, i.e. $y[i] = x[i], i = 1,\dots,T$.

Here we consider consequent procedure of signals filtration is considered. Let's suppose that we have some point $x[i_0]$ and at the same time we know that this point is the first point of useful signal, i.e. $\forall 1 \le i < i_0, y[i] \ne x[i], y[i_0] = x[i_0]$. The possible procedure for detecting of such point will be proposed below. Let's determine the degree of plausibility for some point to useful signal under condition that the point $x[i_0]$ belongs to useful signal as following:

$$l(i,i_0) \triangleq l\{y[i]=x[i] \mid y[i_0]=x[i_0]\} = \\ = -\frac{(x[i]-x[i_0]-M_{i,i_0})^2}{2\sum_{i,i_0}^2} - \ln(\sqrt{2\pi}\sum_{i,i_0}) \quad (1)$$

Here M_{i,i_0} and $\sum_{i,i_0}$ are expressed in formulas through centroid and variance fields correspondingly:

$$M_{i,i_0} = \sum_{k=i_0}^{i} \mu[k] \quad (2)$$

$$\sum_{i,i_0} = \sqrt{\sum_{k=i_0}^{i} \sigma^2[k]} \quad (3)$$

At the time we fix the current point of useful signal the next point is found by maximizing likelihood function (1) with respect to time moments. The set of constructed in such a way points we refer as *admissible points*. The rest points of signal $x[i]$ are considered as noise and ignored. So, the useful signal $y[i]$ is a consequence of admissible points, between which the output signal is completed with the aid of interpolation.

Formula (1) of likelihood function is connected with following. Let us put the input signal as realization of some stochastic process $\varsigma(t)$. In the investigated task we try to cope with significant stochastic noise, which can't be explained from viewpoint of preceding points. Taking this into account it's quite natural to suppose the following idea:

$$\varsigma(t_1)-\varsigma(t_0) = A(t_0,t_1)(\xi(t_1)-\xi(t_0)) + \\ +B(t_0,t_1)+\eta(t_1), \forall 0 \le t_0 < t_1 \le T \quad (4)$$

Here $\xi(t)$ is Wiener process, and $\eta(t)$ is a process with unknown properties, which reflects the presence of noise in signal. Using (4) it is possible to prove that in case of absence of significant noise the differences of input stochastic process, correspondingly centred and normalized, have standard normal distribution. The latter in natural way brings to method of filtration, based on application of maximal likelihood principle. Let we know, that the input signal is not distorted at some point. Then the determination of the next point of useful signal goes with maximization of probability density of normal distribution, which is reflected by formula (1).

For implementation of mentioned filtration algorithm there is a need to estimate centroid and variance fields. The centroid field $\mu[i]$ can be interpreted as our preference concerning the tendency of signal's values changing, and variance field in its turn characterizes our assumptions about local measure of variability of useful signal at some point. For determination of functions values $\mu[i]$ and $\sigma[i]$ it is possible to use our prior knowledge about approximate shape of signal or to use preliminary processing of investigated signal with the aid of low-pass filters, for instance, moving average. Let $z[i]$ is signal obtained by smoothing of the input one. Then centroid and variance fields can be determined with following formulas:

$$\mu[i] = z[i+1]-z[i], i=1,\dots,T-1$$
$$\mu[T]=0$$
$$\sigma[i] = \lambda + \delta\left|\mu[i]\right|, i=1,\dots,T$$

where real parameters λ and δ are fitted in compliance with the particular type of signal and noise degree.

According to formula (1) during looking for the current admissible point there is a need to examine the whole signal, which can be connected with calculation difficulties and doesn't allow to process infinite or on-line signals. Let's denote $A_{i_k}[t] = \max_{i_k < i \le i_k+t} l(i,i_0)$. Then the following fact can be proved:

Theorem. For determination of the current admissible point of signal it is enough to examine t points, where t is minimal number, which satisfies the equation

$$\sum_{i=i_k}^{i_k+t} \sigma^2[i] \ge \frac{\exp(-2A_{i_k}[t])}{2\pi}$$

The determination of initial point of signal

Let's suppose in more detail the problem concerning finding the first admissible point. It is not rare case when the first point of the signal can serve as the first point of useful signal. However, such situations are not excluded, when the group of first points may be noise and the problem of determination of the first admissible point becomes non-trivial. Here a procedure, based on adding one point to input signal with further voting on different starting points, is proposed.

More formally, the procedure for finding the first admissible point of signal is following:

Step A. Choose the search interval of the starting point $[0,T_1]$

Step B. Take as the first admissible point $x[i_0]$ the first point of initial signal $x[1]$

Step C. Add initial signal with the aid of centroid field by point $x[0]=x[i_0]-\sum_{i=1}^{i_0}\mu[i]$ and make filtration on interval $[0,T_1]$ from starting point $x[i_0]$

Step D. Calculate criterion function of obtained filtration $F(i_0)$

Step E. $i_0=i_0+1$. If $i_0>T_1$, then stop, else go to step C.

The point, which minimizes the quality functional, is taken as optimal starting point. As criterion function of filtration from different starting points one can take the number of points, considered as noise. The more points stay for output signal from the current starting point, the greater probability for this point to be admissible.

According to the theorem for looking for the first admissible point it is enough to take T_1 as minimal t, which satisfies the condition:

$$\sum_{i=1}^{t}\sigma^2[i]\geq\frac{\exp(-2A_1[t])}{2\pi}$$

Experimental results

The proposed above technology was applied to analysis of discrete signals during solving the task of drug-addict degree determination using the pupil's reaction on light flare [3]. The necessary data was kindly provided by "IRITECH Inc." On figure 1 the typical pupillogram is showed (the consequence of pupil's changing during 2.5 seconds after stimulating impulse). The Y axis is quotient of pupil's diameter to diameter of iris. The qualitative shape of pupillogram is well-known for specialists. Unfortunately, during capturing the noise often appears, connected with camera's motion-blue, head motion, winking of patient, etc. This leads to sufficient distortion, which must be processed. Figures 2 and 3 demonstrate results of application of probabilistic and low-pass filters to typical noisy signals. Some parts of pupillogram (on the figures they are encircled) are of particular importance for further analysis and their shape should be distorted as less as possible. The figures quite evidently indicate the probabilistic filtration to be more appropriate for the supposed task.

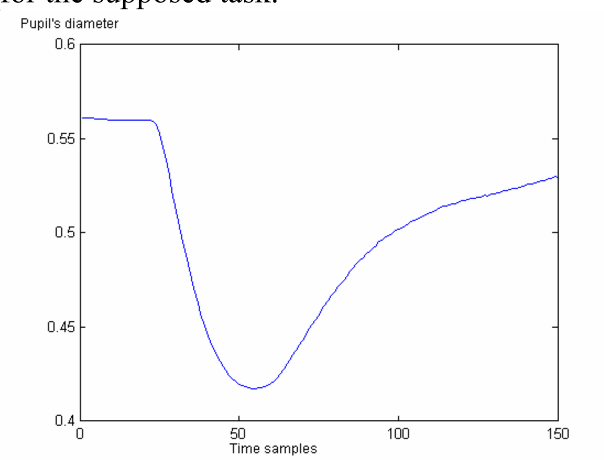

Fig. 1. The common shape of pupillogram.

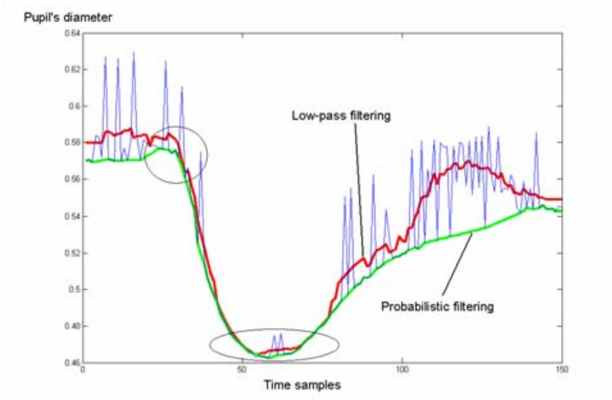

Fig. 2. Application of probabilistic and low-pass linear filters to noisy signal.

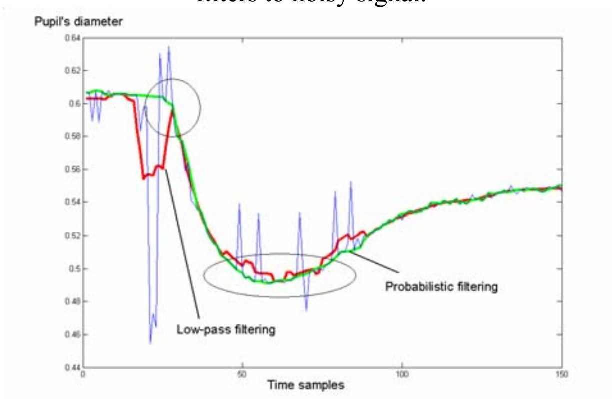

Fig. 3. Application of probabilistic and low-pass linear filters to noisy signal.

Conclusion

In this paper the method of signals filtration, which uses probabilistic approach, is proposed. The algorithm showed its efficiency on the real practical task – processing of pupillograms. The good quality of filter for the tasks similar to investigated one was achieved thanks to the refusal from traditional signals characteristics as impulse and step response. Depending on the value of impulse and step, the corresponding response may change. This allows to remove stochastic noise keeping at the same time local peculiarities of signal.

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